Research Article

Complex Exponential Modulated Quadrature Filter Bank Design Based on Chaos Game Optimization Algorithm

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Abstract

The design approach presented in this paper uses a new metaheuristic method Chaos Game Optimization (CGO) to design a 2-channel quadrature mirror filter (QMF) banks. For this study, finite impulse response (FIR) low-pass prototype filter coefficients are optimized to minimize the cost function. The cost function is formulated as the sum of four terms: the square sum of the errors between this estimated frequency response and the ideal filter response, passband ripple, stopband ripple, and stopband edge frequency error. These parameters are important components of FIR filter prototype design that are often used alone in many studies. To obtain a QMF bank prototype filter, prototype filter modulated with an appropriate complex exponential signal. The produced design results of the proposed method like performance parameters, frequency responses of prototype filters and QMF bank results are presented and compared with earlier reported results. The results show that the presented study has significant improvement in the performance parameters of QMF banks and prototype filter.

Keywords: *Metaheuristic Filter Design, FIR Filter Design, Signal Processing, Optimization, Filter Bank*

1. Introduction

Systems consisting of simple building blocks designed for Digital Signal Processing (DSP) systems shaping the spectrum of data are called digital filters [1]. Digital filters are structural units that amplify or weaken certain components of data in the frequency domain according to the desired output signal. There are two types of digital filters: finite impulse response (FIR) and infinite impulse response (IIR). FIR filters are linear time-invariant (LTI) systems consisting of weighted sums of each input sequence. These systems, which are always stationary, have a linear phase response and, thanks to these advantages, they have a wide usage area [2].

FIR filters have applications for both signals and images [3]. It is quite common to use metaheuristic algorithms in FIR filter design, and there are many studies on this subject [4]. Several studies designed FIR filters based on meta-heuristic algorithms. Genetic algorithm (GA) based [5-8]; Particle Swarm Optimization (PSO) based [7, 9, 10]; Differential Evolution (DE) based [11-13]; Artificial Bee Colony (ABC) based [14-16], Harmony Search (HS) based [17, 18]; Squirrel Search Algorithm (SSA) based [19, 20]. However, there is no study on the design of a FIR filter using Chaos Game Optimization (CGO).

The general FIR filter design using metaheuristic algorithms is as follows.

- Create initial population.
- Calculate fitness values and sort the population accordingly.
- Assign the individual with the best fitness value as the global best.
- Update population.
- If the stop criterion is met stop, if not, return to Step 2.
- Determine the global best as FIR filter coefficients.

Filter banks, which were first introduced by Rothweiler in 1983, are involved in decomposition-composition processes in many areas where digital systems such as audio and video transmission and communication are used [21]. Filter banks separate the data into sub-bands, allowing the data to be transmitted over parallel transmission lines, thus increasing the data transmission rate. Synthesis and analysis banks for the filter banks obtained by modulation of a produced prototype filter with an appropriate cosine function are obtained as given below for the M channel [22].
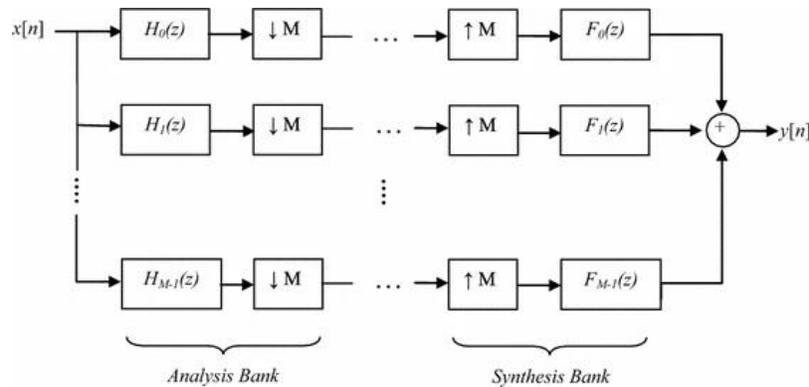
Figure 1 M-Channel Filter Bank Design

There are studies for filter bank design using meta-heuristic algorithms conducted with GA [23-27]; Evolutionary Algorithm (EA) [28], PSO [29-31]; Quantum Behaved PSO [32]; ABC [26, 27, 30, 33, 34]; HS [26, 27]; Hybrid ABC-HA [26]; Hybrid GSA–ABC [26]; Hybrid GSA–HSA [26], Hybrid PSO–ABC [35]; Cuckoo Search Algorithm (CSA) [30] in the literature.

The Chaos Game Optimization (CGA) algorithm, which was introduced by Talatahari et al. in 2020, is a fairly new algorithm and studies with this algorithm are very limited. The CGO algorithm is based on arranging fractals according to the idea of chaos game and arranging fractals according to their self-similarity. Chaos theory states that despite the randomness found in dynamical systems, there are some fundamental patterns in the operation of these systems, such as similar cycles, repetitive shapes, fractals, and multiple subsystems, and thus they can be described as self-similar. Mathematically, a fractal is a subset of Euclidean space in which a given geometric form is repeated at various intervals. Chaos theory states that the current states of dynamical systems can affect the future states of the system, but their approximate current states cannot roughly describe the next state of the system. The chaos game in mathematics is a methodology of creating fractals at that point using a random starting point. The purpose of this is to iteratively produce similar shapes at different scales at successive points [36].

The best example of the CGO approach is to construct the Sierpinski triangle. Accordingly, initial three candidate vertices are determined to form the main shape and a fractal triangle is formed. Candidate vertices are highlighted in red, green and blue. Then, a dice with 2 red faces, 2 green faces and 2 blue faces are thrown and a new fractal is created from there by moving the corner points towards the relevant color of the starting fractal according to the color of the incoming dice. When the dice are rolled enough, the Sierpinski triangle reaches its final shape. The Sierpinski triangle is given below in different scales [37].
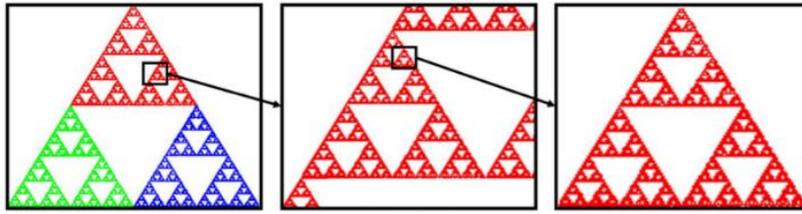
Figure 2 Sierpinski Triangle

In this study, a complex exponential modulated filter bank was designed using the chaos game optimization algorithm as a new approach.

The updated pseudo code of the CGO algorithm used for the FIR filter bank design is as follows.

- Set parameters for CGO.
- Set parameters for ideal filter response.
- Create a population.
- Calculate and sort the fitness values of the population.
- Identify the individual with the best fitness value as the best.
- Update the population.
- If the stopping criterion is met, stop and go to the next step; if not, go to Step 4.
- Determine the individual with the best fitness value as the FIR filter coefficients.
- Build the filter bank by modulating the obtained coefficients.

2. Materials and Methods

In this section, it is explained how to create a 2-channel quadrature mirror filter (QMF) bank from the prototype filters with the proposed cost function using the CGO algorithm. Performance parameters for the designed prototype filters and filter banks are explained in detail. All studies were implemented on a virtual machine with Intel Xeon E5-2695 2.4 GHz dual processor CPU and 36 GB of RAM.

2.1. Prototype Filter Design

The CGO algorithm was used while creating the prototype filter. A prototype filter design was created by making some changes to the method presented by Talatahari. First, an ideal filter response is created. While creating the ideal filter structure, the number of channels (M) and the roll of factor (RF) parameters that adjust the amount of interference in the subbands of the filter bank by adjusting the passband width are determined. For a 2-channel filter bank design, the pass and stop band normalized cut-off frequencies are determined as follows. When the RF value is less than 1, the amount of interference decreases, while if it is greater than 1, the amount of interference increases [21, 38, 39].

First, the number of channels is determined, in this study it is set as 2. In this way, passband normalized cut-off frequency (wp) is indirectly determined as 0.5 pi/rad. After this stopband normalized cut-off frequency (ws) is set as variable according to the RF value [30]. The sample ideal filter responses obtained for different values of these parameters are shown in Figures 3-7.

Table 1 Desired Filter Response

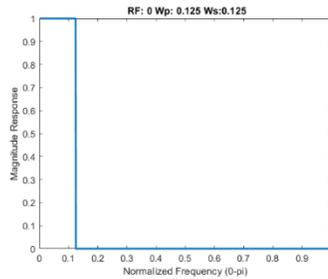|	
$ws = \dfrac{(1 + RF) * \pi}{2 * M}$	$wp = \dfrac{\pi}{2 * M}$
$H_{desired}(e^{jw}) = 1$	$w \leq w_p$
$H_{desired}(e^{jw}) = 0 \leq H_{desired} \leq 1$ linearly spaced vector	$wp \leq w \ \& \ w \leq ws$
$H_{desired}(e^{jw}) = 0$	$w \geq w_s$

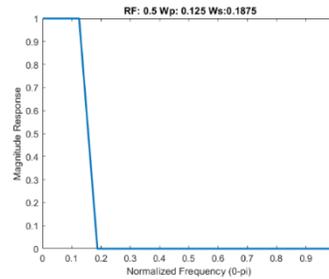*Figure 3 Ideal Filter Response-1*
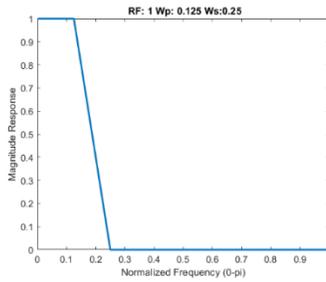

Figure 4 Ideal Filter Response-2
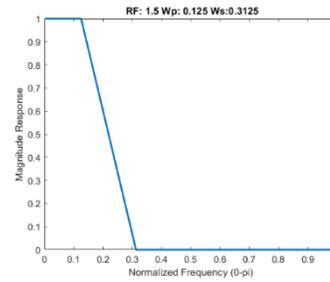
Figure 5 Ideal Filter Response-3
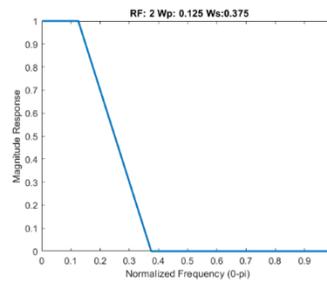

Figure 6 Ideal Filter Response-4

Figure 7 Ideal Filter Response-5

2.2. Cost Function

During the filter design using a metaheuristic algorithm, the bk coefficients of the FIR filter were determined as the parameters that were tried to be optimized. To optimize the bk coefficients, each population member created in the CGO was considered as a bk coefficient solution. Then, the frequency response of the filter was calculated using the bk coefficients. The square sum of the errors between this obtained frequency response and the ideal filter response (MSE), the passband maximum ripple (pr), the stopband maximum ripple (sr) and the stopband edge frequency error (est) are calculated using the following equations. It was attempted to minimize the sum of these calculated values. If a peak is not found for the passband, the maximum point in the frequency response before the wp in the estimated filter response is the passband maximum ripple; If a peak is not found for the stopband, the maximum point in the frequency response after the ws in the estimated filter response is set as the stopband maximum ripple. Cost function parameters and formulation are given in Table 2 and Figure 8.

Table 2 Cost Function and Formulation

The square sum of the errors between this estimated frequency response and the ideal filter response	$mse = \sum (H_{desired}(e^{jw}) - H_{estimated}(e^{jw}))^2$
Passband ripple	$pr = max\left\{Peaks\left(H(e^{jw})\right)\right\} w \le w_p$
Stopband ripple	$sr = max\left\{Peaks\left(-H(e^{jw})\right)\right\} w \ge w_s$
Stopband edge frequency error	$est = \left(H(e^{jw}) \, w = w_s\right)$
Total Error	$error = mse + pr + sr + est$
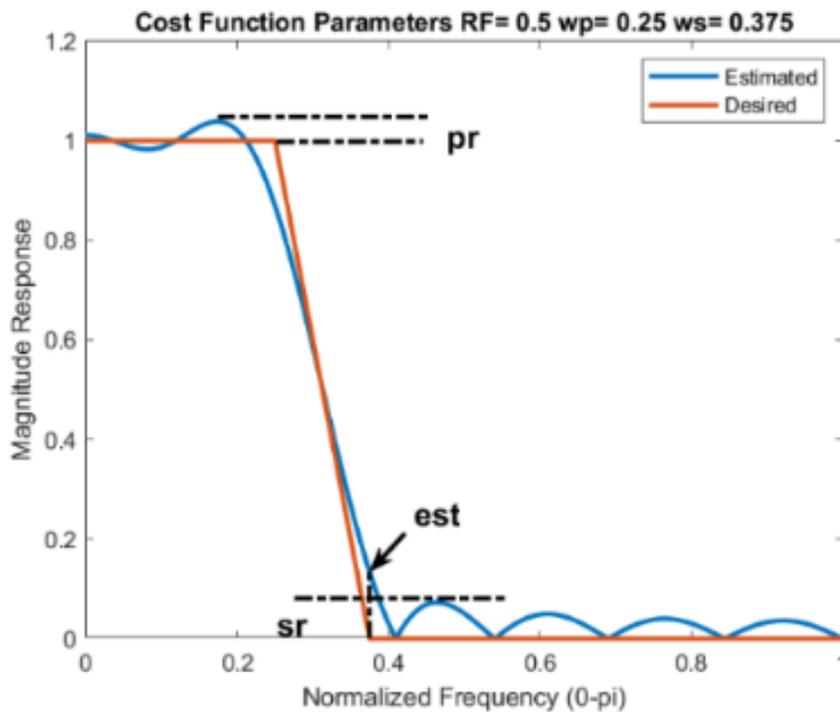
Figure 8 Cost Function Parameters

2.3. Filter Bank Design

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After the prototype filter was created, the modulation process was carried out with the equations given in Table 3. An example QMF Bank filter response is given in Figure 9.

Table 3 QMF Bank Modulation

QMF Bank Channel-1	QMF Bank Channel-2
$h_1[n] = h[n] * e^{i*2*(wp/2)*pi*(1:Degree)}$	$h_2[n] = h[n] * e^{i*2*(wp/2)*pi*2*(1:Degree)}$

Figure 9 Example Filter Bank

2.4. Performance Parameters

The performance parameters and their formulations obtained for the prototype filter and filter banks are selected from the parameters frequently used in the literature are given inTable 4.

Table 4 Performance Parameters

Total Iteration Time

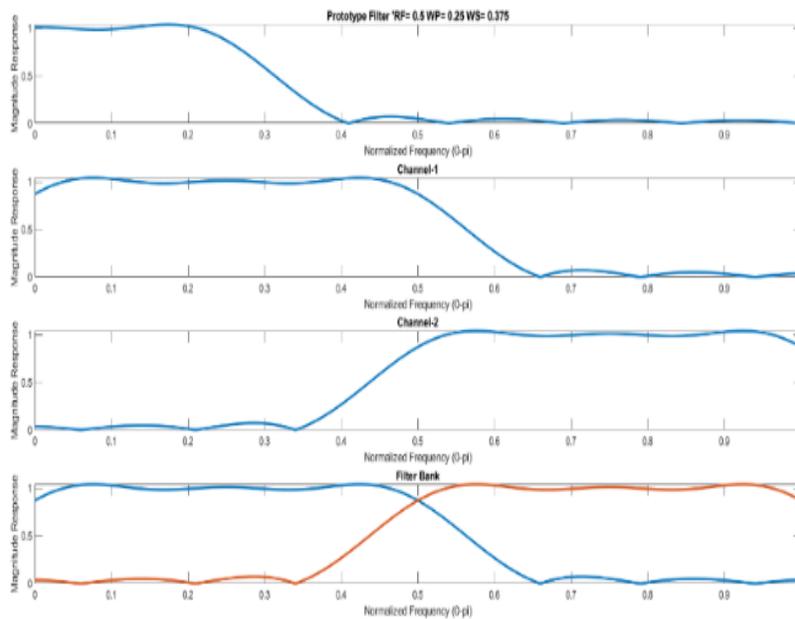| Passband Error	$Ep = \dfrac{1}{\pi}\sum\limits_{0}^{wp}[H(e^{j0}) - H(e^{jw})]^2$
Transitionband Error	$Et = [(H\left(e^{j\frac{1}{\pi}}\right) - (0.707H(e^{j0})]^2$
Stopband Error	$Es = \dfrac{1}{\pi}\sum\limits_{ws}^{\pi}[H(e^{jw}]^2$
Correlation Between Ideal Filter and Prototype Filter	$Corr = 1 - (correlation\{H_{desired}(e^{jw}), H_{estimated}(e^{jw})\})$
The Square Sum of the Errors Between Obtained Frequency Response and the Ideal Filter Response	$mse = \sum(H_{desired}(e^{jw}) - H_{estimated}(e^{jw}))^2$
Passband Ripple	$pr = max\left\{Peaks\left(H(e^{jw})\right)\right\} w \le w_p$
Stopband Ripple	$sr = max\left\{Peaks\left(-H(e^{jw})\right)\right\} w \ge w_s$
Stopband First Lobe Attenuation	$Asl = -20 * log_{10}\left(\dfrac{max\left\{Peaks\left(H(e^{jw})\right)\right\}}{w \ge w_s}\right)$
Stopband Edge Frequency Attenuation	$As = -20 * log_{10}(H(e^{jw}) w = w_s)$
Filter Bank Peak Reconstruction Error	$max\left\{10 * log_{10}\left(\sum\limits_{k=1}^{M} H_i(e^{jw})^2\right)\right\}$
Filter Bank Peak Aliasing Error	$max\left\{\sqrt{\left(\sum\limits_{k=1}^{M} H_i(e^{jw})^2\right)}\right\}$
Filter Bank Peak To Peak Reconstruction Error	$max\left\{\sqrt{\left(\sum\limits_{k=1}^{M} H_i(e^{jw})^2\right)}\right\}$ $-min\left\{\sqrt{\left(\sum\limits_{k=1}^{M} H_i(e^{jw})^2\right)}\right\}$

2.5. Simulation Results

The performance parameters of all prototype filters obtained using the CGO algorithm are given in supplementary material Performance parameters of prototype filters that give the best results individually according to performance results are given in Table 5 and filter banks and prototype filters obtained from these prototype filters are given in Figures 10-15.

Table 5 Performance Results

|	N	RF	wp	ws	Ep	Et	Es	pr	sr	Asl	As	MSE	Corr	pre	pae	p2pre
Ep	48,00	2,00	0,25	0,75	0,0006	0,002	0,00727	0,00007301	0,0072	42,90	90,38	0,027	0,00007	2,94	1,40	0,41
Et	22,00	2,00	0,25	0,75	0,0055	0,001	0,00368	0,00080114	0,0049	46,26	38,88	0,024	0,00006	2,83	1,39	0,41
Es	32,00	2,00	0,25	0,75	0,0039	0,002	0,00108	0,00281871	0,0029	50,71	50,22	0,018	0,00005	2,94	1,40	0,41
pr	16,00	2,00	0,25	0,75	0,0041	0,001	0,00516	0,00000002	0,0056	45,07	35,79	0,024	0,00006	2,83	1,39	0,41
sr	32,00	2,00	0,25	0,75	0,0039	0,002	0,00108	0,00281871	0,0029	50,71	50,22	0,018	0,00005	2,94	1,40	0,41
Asl	32,00	2,00	0,25	0,75	0,0039	0,002	0,00108	0,00281871	0,0029	50,71	50,22	0,018	0,00005	2,94	1,40	0,41
As	48,00	2,00	0,25	0,75	0,0006	0,002	0,00727	0,00007301	0,0072	42,90	90,38	0,027	0,00007	2,94	1,40	0,41
MSE	48,00	1,50	0,25	0,63	0,0007	0,016	0,00604	0,00002649	0,0064	43,87	53,83	0,008	0,00001	2,94	1,40	0,41
Corr	48,00	1,50	0,25	0,63	0,0007	0,016	0,00604	0,00002649	0,0064	43,87	53,83	0,008	0,00001	2,94	1,40	0,41
pre	48,00	0,00	0,25	0,25	1,6683	0,081	2,21822	0,05915343	0,1132	18,92	6,37	4,135	0,00865	0,55	1,07	0,58
pae	48,00	0,00	0,25	0,25	1,6683	0,081	2,21822	0,05915343	0,1132	18,92	6,37	4,135	0,00865	0,55	1,07	0,58
p2pre	12,00	0,50	0,25	0,38	0,2617	0,077	0,95195	0,03822591	0,0725	22,79	17,47	1,830	0,00260	1,80	1,23	0,36
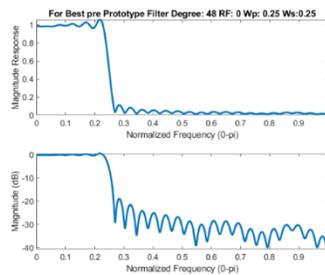
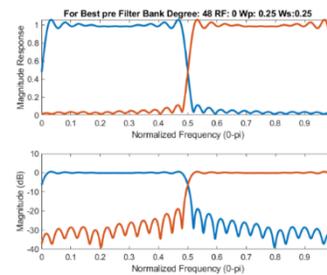*Figure 10 Best pre Prototype Filter*
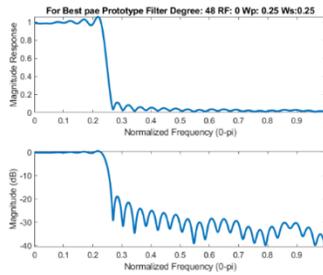

Figure 11 Best pre QMF Bank
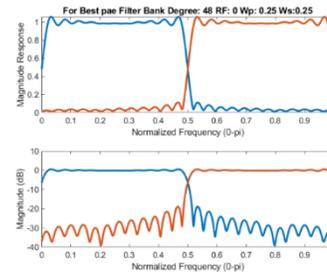
Figure 12 Best pae Prototype Filter
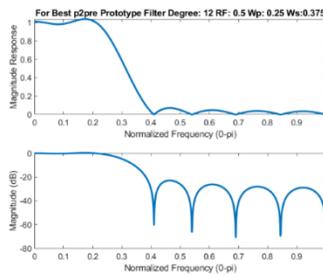

Figure 13 Best pae QMF Bank

Figure 14 Best p2pre Prototype Filter
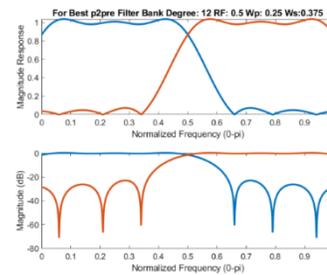
Figure 15 Best p2pre Prototype Filter

3. Discussion

To be able to compare with the literature; the prototype filter length N is equal to 32, ws= 0.6 pi, and wp=0.4 pi is set. CGO algorithm is run and the following 32 optimal filter coefficients are obtained and given in Table 6. Filter bank obtained from prototype filter by converting from low pass prototype filter to high pass filter without modulation is given in Figure 16 and Figure 17.

Table 6 Coefficients for N=32 wp=0.4 pi ws=0.6 pi

h[0]	-0.00733232769509882	h[16]	-0.205216800572325
h[1]	0.00202234412671953	h[17]	-0.184094702804906
h[2]	0.00794135521073253	h[18]	0.0191331905116305
h[3]	-0.0114098293750996	h[19]	0.127101871037846
h[4]	-0.0209252303563760	h[20]	0.0211209318115592

|			
h[5]	0.0123886438404877	h[21]	-0.184031872690335
h[6]	0.0211954315684602	h[22]	-0.328865434731056
h[7]	-0.0333334727659447	h[23]	-0.329971725897613
h[8]	-0.0478182849362887	h[24]	-0.206589727305304
h[9]	0.0335878900117232	h[25]	-0.0560936926820710
h[10]	0.0556221961504517	h[26]	0.0407608278200729
h[11]	-0.0661725080316661	h[27]	0.0825936751212010
h[12]	-0.127188556820167	h[28]	0.0967636748504352
h[13]	0.0152937797534578	h[29]	0.0836828737592829
h[14]	0.134702042284019	h[30]	0.0474822804005983
h[15]	0.00103759475409229	h[31]	0.0153011319867537
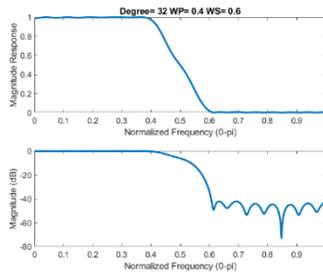
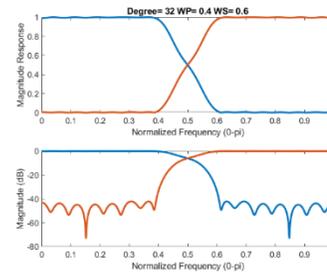*Figure 16 Prototype Filter without Modulation*

Figure 17 QMF Bank without Modulation

In addition, the design parameters of the filter bank obtained by the proposed complex exponential modulation method are as follows; N is equal to 32, wp=0.25 pi, RF=1.5 and relatively ws=0.625 pi. For the proposed method filter coefficients are given in Table 7. To perform QMF bank design, RF is determined as 1.5 [30]. The filter bank obtained from the prototype filter by the proposed complex exponential modulation method is given in Figure 18 and Figure 19.

Table 7 Coefficients for N=32 wp=0.25 pi RF=1.5 ws=0.625 pi

h[0]	-0,00945751931367619	h[16]	-0,26147795680066400
h[1]	-0,00717635928542794	h[17]	-0,26559668614005800
h[2]	0,02038953921171140	h[18]	-0,14993837542822500
h[3]	0,03780089479412950	h[19]	-0,09578908113094580
h[4]	0,01447794405780110	h[20]	-0,10075412683110000
h[5]	-0,02323881690430300	h[21]	-0,09735298375831200
h[6]	-0,03973726847080890	h[22]	-0,09595931495554310
h[7]	-0,04707368400527060	h[23]	-0,11053497631368400
h[8]	-0,04293349209342300	h[24]	-0,10908285701746200

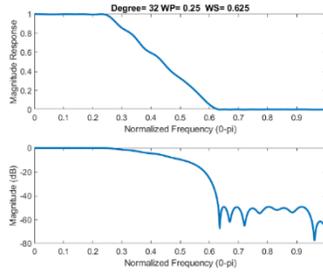| h[9]	-0,01854485200354470	h[25]	-0,07260943198703210
h[10]	-0,00617735807269981	h[26]	-0,02742162001311890
h[11]	0,00450552798491457	h[27]	-0,00489974593787194
h[12]	0,10486638555063600	h[28]	-0,01075456098552490
h[13]	0,25723607936091200	h[29]	-0,02436988846557280
h[14]	0,23606205577851000	h[30]	-0,02267571661952870
h[15]	-0,02377528351342570	h[31]	-0,00863766150764813

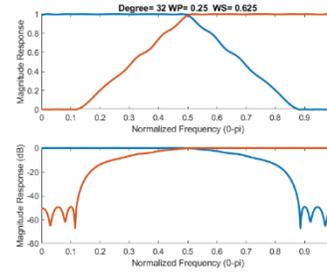*Figure 18 Prototype Filter with Modulation*

Figure 19 QMF Bank with Modulation

A comparison of the performance with previous state-of-the-art methods is presented in Table 8.

Table 8 Performance Comparison of Proposed CGO Method with Different Algorithms

	Ep	Et	Es	pr	sr	As	Asl	MSE	Corr	pre	pae	p2pre
[40]	0,000047	0,085739	**0,003057**	0,000692	0,006168	34,870290	44,197429	4,747669	0,008899	0,050830	1,005869	0,006749
[41]	0,000029	0,085770	0,003112	0,000534	0,005953	34,558430	44,505305	4,743008	0,008886	0,052667	1,006082	0,006814
[42]	70,336765	0,499847	0,006660	0,414970	0,008708	30,239978	41,201290	109,114475	0,008842	3,064900	1,423131	0,009811
[43]	**0,000039**	0,085772	0,003159	0,000622	0,006761	36,061045	43,399183	4,758001	0,008928	0,051617	1,005960	0,006831
[44]	0,000459	0,085406	0,021158	0,001750	0,011226	39,000789	38,995257	5,520960	0,010682	0,061326	1,007085	0,008798
[45]	34,431019	0,339260	0,005599	0,290497	0,007915	31,224525	42,031414	59,338560	0,008892	2,284451	1,300836	0,011690
[46]	48,166490	0,403940	0,005858	0,343279	0,007811	31,179514	42,146101	78,475099	0,008804	2,608171	1,350232	0,007967
[47]	33,964649	0,332991	0,199453	0,308871	0,044365	**40,681659**	27,059281	58,201342	0,008029	2,337945	1,308872	0,035096

[48]	0,000 043	0,085 672	0,003 104	**0,000 300**	**0,005 901**	34,43 5257	**44,58 1216**	4,732 480	0,008 878	0,046 671	1,005 388	**0,006 246**
Proposed without modulation	0,005 677	**0,084 426**	0,011 969	0,001 371	0,007 813	33,55 9577	42,14 3128	**0,048 393**	**0,000 092**	**0,011 967**	**1,001 379**	0,292 808
Proposed with modulation	0,001 063	**0,015 829**	**0,002 442**	**0,000 001**	**0,003 462**	**41,24 1744**	**49,21 2998**	**0,032 983**	**0,000 083**	2,901 888	1,396 672	0,409 071

When Table 8 is examined, without the modulation method five of the twelve total performance parameters have better results than the others; with the modulation method eight of the twelve total performance parameters have better results than the others.

4. Conclusion

In this study, a 2-channel QMF bank with complex exponential modulation was designed using the CGO algorithm. The developed algorithm was run at filter degree for 12,16,22,24,32 and 48; roll of factor values of 0,0.5,1.0,1.5 and 2 for different stopband edge frequencies. In total 30 different prototype filters and filter banks were designed. The proposed cost function is minimized for low-pass filters by using the CGO algorithm. The performance parameters of the study using the CGO algorithm were compared with PSO, DE and other well-known QMF bank design problems. When the simulation results were examined, it was observed that it made improvements in performance parameters compared to other state-of-the-art studies. It also appeared to be efficient in high-order prototype filter designs.

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