Research Article

Dynamic Parameter Estimation for a Washing Machine

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Abstract

In this study, an analytical concept is explored in the simulation environment, and an unbalanced load detection technique is examined to determine its applicability. To continue the development process, a mathematical model for a horizontal washer is developed to comprehend the characteristics of an unbalanced load with respect to various sizes of balanced loads. The mathematical model is then divided into a regressor vector and an estimator vector. Finally, estimates of the unbalanced load, the unbalance location, and the rotational moment of inertia are compared to their references in the simulation environment.

Keywords: Washing machine, Dynamic parameters, Parameter estimation, Unbalanced Mass, Recursive least square

1. Introduction

The imbalance detection of a washer during the washing cycle is a critical process to reduce the structural vibration and the failure of the mechanical components in a short period of time. To understand the effect of the vibration, authors constructed mathematical models for modal analysis and investigated it experimentally (Conrad1995, Agnani2008, Lim2010, Nygaards2012, Bascetta2012). From the mathematical background, researches were focused on detecting the unbalanced mass precisely using the neural network by measuring the vibration amplitude (Yuan2007, Yorukouglu2012) and the current ripple (Jung2015).

Unlike other references, a less cost-effective method was proposed to detect the unbalanced load using an unbalance observer composed of an unbalance torque observer and an estimator with filters at constant speed (Kim2009). By detecting the amount of unbalance, a ball balancer and a hydraulic damper were considered (Bae2002, Chen2017) to decrease its effect during the spinning phase.

In addition, adding a semi-active damper was proposed to reduce the vibration induced by the unbalanced weight in an adaptive control loop (Spelta2009). In the perspective of industrial applications, however, the above solutions can be applied only to the high-end product due to the cost effect. The typical imbalance estimation process in commercial products is carried out at the low speed ranges in a steady-state condition and the distribution process is applied if the detected unbalanced mass is not allowable to get into the spinning phase for the water extraction from the laundry.

One of the major issues of the conventional estimation technique in the home-appliance sector is that the unbalanced mass is treated as the static laundry at a steady-state speed. However, during the drum motion at a variety of the speed range, not only the magnitude of the unbalanced mass varies over time until the water is removed from the laundry properly, but also the speed is not constant even in a steady-state condition due to the unbalanced load. This may lead to an extra filtering process to remove the speed or the torque ripple (Cao2018). In addition, estimating the mechanical parameters is performed separately regarding the priority among them. Therefore, a more rigorous method is required to capture this behavior simultaneously than the current algorithm in the existing product.

The main objective of this study is to provide both the mathematical modeling for a horizontal washer and the estimation framework for the mechanical parameters simultaneously in the closed-loop system. The proposed method is cost effective and applicable to washing machines regardless of the drum capacity and the laundry amount.

From the industrial point of view, this method has the potential to minimize the cost to obtain the maximum productivity for the highest satisfaction regarding the end-user.

2. Materials and Methods

In this section, the dynamics of a washing machine will be studied in the presence of the unbalanced mass around the drum. By simulating the proposed algorithm, our aim is to estimate the varying mechanical parameters such as total inertia, and the magnitude of the unbalanced mass, which are decision-makers during the washing cycles. The RLS estimation process is performed in the control loops in Fig. 1. The washing machine is operated by two current PI controllers in the inner-loop, while the drum speed is controlled by a PI controller in the outer-loop. Especially, detecting the size of an

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unbalanced load is critical before the motor runs at the higher speed region. Thus, in the presence of the unbalanced mass around the drum wall, the redistribution algorithm can be applied to make the balanced load distribution if the unbalance mass is not acceptable at a given speed plateau.

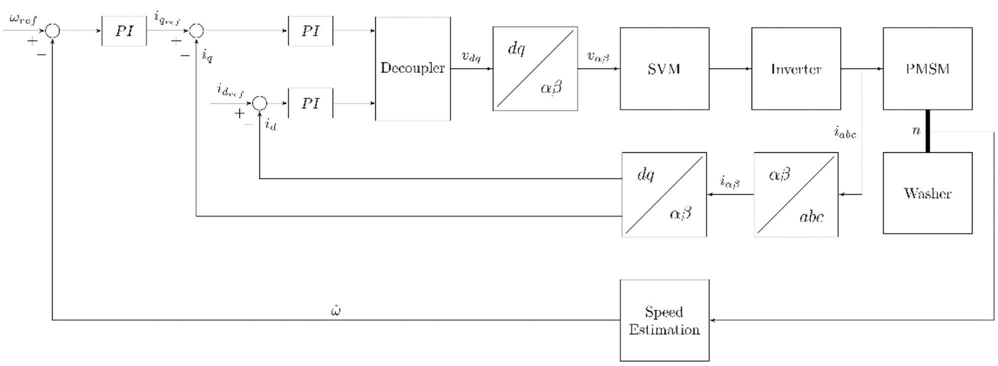

Figure 1 The Control Scheme of a IPMSM

In Fig. 1, a washing machine is operated by an interior permanent magnet motor (IPMSM) via a belt-transmission from the motor to the drum shaft and its ratio is given as n. The mathematical models for the IPMSM drive are given in (1)-(3).

$$v_d = Ri_d + L_d\frac{di_d}{dt} - \omega_r L_q i_q \quad (1)$$

$$v_q = Ri_q + L_q\frac{di_q}{dt} + \omega_r(\psi_{pm} + L_d i_d) \quad (2)$$

$$\tau_e = \frac{3p}{4}\left[\psi_{pm} i_q + (L_d - L_q) i_d i_q\right] \quad (3)$$

where v_d and v_q are the stator voltage on the dq-frame, R is the stator resistance, L_d and L_q are the stator inductances, ω_r is the rotor speeds, ψ_{pm} is the flux linkage, p is the number of poles and τ_e is the electric torque. In addition, the rotor angular speed will be n times faster than the one of a drum (ω_d). These basic IPMSM equations are used to construct the controllers via coordinate transformation and the space vector modulation (SVM) in a closed-loop system.

2.1.Washer Dynamics

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In order to model the physical effect of laundry behavior in the drum, we treated the laundry as balanced and unbalanced loads in Fig. 2. To proceed further study, there are several assumptions as below,

Assumptions:

1) The balanced loads are identical as well as the distance from the drum center to the load center (i.e., $m_1 = m_2 = m_3, r_1 = r_2 = r_3$).
    a) a number of the balanced loads mimic the distributed balanced load in the drum. The greater number of the balanced load, the more accurate geometry can be obtained.
    b) the position deviation between the balanced loads are evenly fixed as $\theta_1 = \theta_2 = \theta_3 = 2\pi/3$ rad.

2) The unbalanced load (m_{ub}) is located on top of a mass m_1 and its distance is defined as r_{ub}.
    a) the position of the unbalanced load can be defined by $\alpha = \theta_d + \beta$: assume that the unbalanced load has a fixed offset (β) from the drum position (θ_d).
3) For the simple derivation of a mathematical modeling, it is assumed that β is zero degree but we will compensate for that in the final mathematical model.

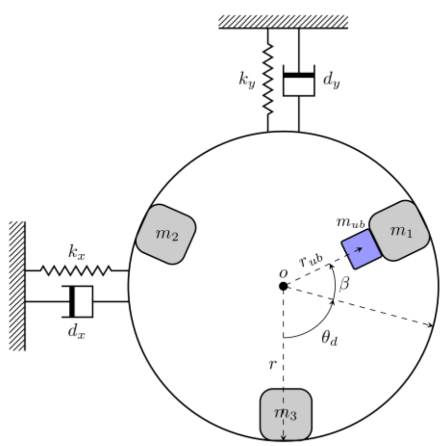

Figure 2 Washer Diagram with the Fixed Balanced and Unbalanced Loads

Consider Figs. 2 to derive the dynamics of the transnational and rotational motion for a horizontal washing machine.

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$$\left(M + m_{ub} + \sum_{i=1}^{3} m_i\right)\ddot{x} + d_x\dot{x} + k_x x$$
$$= m_1 r_1 \dot{\theta}_d^2 \sin\theta_d + m_1 r_1 \ddot{\theta}_d \cos\theta_d$$
$$+ m_2 r_2\, \dot{\theta}_d^2 \sin(\theta_d + 2\pi/3) + m_2 r_2 \ddot{\theta}_d \cos(\theta_d + 2\pi/3)$$
$$+ m_3 r_3 \dot{\theta}_d^2 \sin(\theta_d + 4\pi/3) + m_3 r_3 \ddot{\theta}_d \cos(\theta_d + 4\pi/3)$$
$$+ m_{ub} r_{ub} \dot{\theta}_d^2 \sin\theta_d + m_{ub} r_{ub} \ddot{\theta}_d \cos\theta_d \quad (4)$$

$$\left(M + m_{ub} + \sum_{i=1}^{3} m_i\right)\ddot{y} + d_y\dot{y} + k_y y$$
$$= m_1 r_1 \dot{\theta}_d^2 \cos\theta_d - m_1 r_1 \ddot{\theta}_d \sin\theta_d$$
$$+ m_2 r_2\, \dot{\theta}_d^2 \cos(\theta_d + 2\pi/3) - m_2 r_2 \ddot{\theta}_d \sin(\theta_d + 2\pi/3)$$
$$+ m_3 r_3 \dot{\theta}_d^2 \cos(\theta_d + 4\pi/3) - m_3 r_3 \ddot{\theta}_d \sin(\theta_d + 4\pi/3)$$
$$+ m_{ub} r_{ub} \dot{\theta}_d^2 \cos\theta_d - m_{ub} r_{ub} \ddot{\theta}_d \sin\theta_d \quad (5)$$

where M is the total mass of the mechanical components composed of the drum, tub, motor, and shaft of a washing machine.

Now, consider that the first balanced load (m_1) has a fixed offset (β) from the drum angular position (θ_d) and the unbalanced load is located on top of it. In Fig. 2, the angles between the balanced loads are defined as $2\pi/3$ rad. Then, we can write the rotational motion of a washer in (6).

$$J_t\dot{\omega}_d = \tau - b_t\omega_d - c - m_{ub} r_{ub}[(g + \ddot{y})\sin\theta_d - \ddot{x}\cos\theta_d]$$
$$- m_1 r_1[(g + \ddot{y})\sin\theta_d - \ddot{x}\cos\theta_d]$$
$$- m_2 r_2\left[(g + \ddot{y})\sin(\theta_d + 2\pi/3) - \ddot{x}\cos(\theta_d + \frac{2\pi}{3})\right]$$
$$- m_3 r_3[(g + \ddot{y})\sin(\theta_d + 4\pi/3) - \ddot{x}\cos(\theta_d + 4\pi/3)] \quad (6)$$

where J_t, b_t, c, m_i, and r_i are the total inertia, the total viscous friction coefficient, the Coulomb torque, locally distributed mass in the drum, and the distance from the center of drum to the center of mass (m_i). From (6), the total inertia is composed of the drum inertia (J_d), the motor inertia (J_m), the inertia from the balanced load (J_b), and the inertia induced by the unbalanced mass ($m_{ub} r_{ub}$). Extra forces and momentums induced by the balanced loads in the drum cancel each other out, and the simplified dynamics from (4)-(6) can be expressed by adding the fixed unbalance location in (7)-(9).

$$\left(M + m_{ub} + \sum_{i=1}^{3} m_i\right)\ddot{x} + d_x\dot{x} + k_x x$$
$$= m_{ub} r_{ub} \dot{\theta}_d^2 \sin(\theta_d + \beta) + m_{ub} r_{ub} \ddot{\theta}_d \cos(\theta_d + \beta) \quad (7)$$

$$\left(M+m_{ub}+\sum_{i=1}^{3}m_i\right)\ddot{y}+d_x\dot{y}+k_y y = m_{ub}r_{ub}\dot{\theta}_d^2\cos(\theta_d+\beta)-m_{ub}r_{ub}\ddot{\theta}_d\sin(\theta_d+\beta) \quad (8)$$

$$J_t\dot{\omega}_d+b_t\omega_d+c=\tau-m_{ub}r_{ub}[(g+\ddot{y})\sin(\theta_d+\beta)-\ddot{x}\cos(\theta_d+\beta)] \quad (9)$$

Since the magnitude of the translational acceleration at low angular speed is less compared to the gravitational acceleration, we only consider the gravitational acceleration (g). Then, equation (9) can be re-written as (10).

$$J_t\dot{\omega}_d+b_t\omega_d+c+m_{ub}r_{ub}g\sin(\theta_d+\beta)=\tau \quad (10)$$

2.2.Parameter Estimation using RLS

In this section, we study how to use the measured signals for identifying the physical quantities. Therefore, a recursive least square (RLS) method is employed for the estimation of mechanical parameters. To decrease the influence of past signals during the current parameter estimation, a cost function, $V(\Theta,k)$, with an exponential forgetting factor (λ) should be minimized in (11).

$$\min V(\Theta,k)=\frac{1}{2}\sum_{i=1}^{k}\lambda^{k-i}[y(i)-\Phi^T(i)\Theta(i)]^2 \quad (11)$$

Then, the generalized RLS algorithm follows the steps from (12) to (16).

$$\hat{y}(k)=\Phi^T(k)\hat{\Theta}(k-1) \quad (12)$$

$$\epsilon(k)=y(k)-\hat{y}(k) \quad (13)$$

$$K(k)=P(k-1)\Phi(k)[\lambda I+\Phi^T(k)P(k-1)\Phi(k)]^{-1} \quad (14)$$

$$P(k) = \frac{[I - K(k)\Phi^T(k)]P(k-1)}{\lambda} \tag{15}$$

$$\widehat{\Theta}(k) = \widehat{\Theta}(k-1) + K(k)\epsilon(k) \tag{16}$$

where $y(k)$ is the output, $\widehat{\Theta}(k)$ and $\widehat{\Theta}(k-1)$ are the estimates of current and previous steps, $\Phi(k)$ is the feedback by the measurement, λ is the forgetting factor between 0 and 1 to improve the tracking capability, $\epsilon(k)$ is the estimation error, $P(k)$ is the covariance matrix, and $K(k)$ is the correction gain matrix.

For practical use of the RLS, we assume that the unbalanced mass varies during the simulation process to mimic the actual washing machine characteristics. This requires the different weight on the current estimate to reduce the error between the actual and the estimate of the mechanical parameters (Raol2004).

In order to express the rotational motion as the form of the regressor and the parameter vectors, equation (10) can be reshaped in an acceleration form in (17).

$$\dot{\omega}_d = \frac{1}{J_t}[\tau - b_t\omega_d - c - m_{ub}r_{ub}g(\sin\theta_d \cos\beta + \cos\theta_d \sin\beta)] \tag{17}$$

For the implementation into the micro-controller, a discretization using a zero-order hold (ZOH) with a fixed time step ($\boldsymbol{T_s}$) is applied for the angular speed estimation.

$$\begin{aligned}\widehat{\omega}_d(k) = -\Omega\widehat{\omega}_d(k-1) \\ + \Gamma(\tau(k-1) - c - m_{ub}r_{ub}g\sin\theta_d(k-1)\cos\beta \\ - m_{ub}r_{ub}\cos\theta_d(k-1)\sin\beta)\end{aligned} \tag{18}$$

where $\Gamma = (1+\Omega)/b_t$ and $\Omega = -\exp(-b_t T_s/J_t)$

Then, we can write (18) as the form of (12). Therefore, the speed estimation can be obtained in (19).

$$\widehat{\omega}_d = \Phi^T \widehat{\Theta} \tag{19}$$

where

$$\Phi^T = [\omega_d(k-1) \quad \tau(k-1) \quad 1 \quad \sin\theta_d(k-1) \quad \cos\theta_d(k-1)]$$

$$\widehat{\Theta} = \begin{bmatrix} -\Omega \\ \Gamma \\ -\Gamma c \\ -\Gamma m_{ub} r_{ub} g \cos\beta \\ -\Gamma m_{ub} r_{ub} g \sin\beta \end{bmatrix}$$

By manipulating the estimation vector at each time step, mechanical parameters are computed in (20).

$$\hat{b}_t = \frac{1-\widehat{\Theta}_1}{\widehat{\Theta}_2}, \hat{J}_t = -\frac{\hat{b}_t T_s}{\ln\widehat{\Theta}_1}, \hat{c} = -\frac{\widehat{\Theta}_3}{\widehat{\Theta}_2}, \widehat{m}_{ub} = \frac{norm(\mu_1,\mu_2)}{g r_{ub}}, \hat{\beta} = \sin^{-1}\left(\frac{\mu_2}{m_{ub} r_{ub} g}\right) \tag{20}$$

where $\mu_1 = -\widehat{\Theta}_4/\widehat{\Theta}_2$ and $\mu_2 = -\widehat{\Theta}_5/\widehat{\Theta}_2$

To improve the estimation accuracy, we added a periodic signal in the feedback loop.

3. Simulation Results

Numerical simulations using MATLAB/SIMULINK R2017b for the parameter estimation were performed based on the specification in Table 1. The static parameters indicate the nominal values without any loads in the drum, while the non-valued parameters are variable over time.

Table 1 Mechanical Parameters

$M = 39\ kg$	m_{ub}	J_t	$k_x = k_y = 5000\ N/m$
$m_{bal} = 6\ kg$	r_{ub}	$b_t = 0.002\ Nm/(rad/s)$	$d_x = d_y = 110\ N/(m/s)$
$m_i = m_{bal}/3$	$r_i = 0.22\ m$	$c = 0.0502\ Nm$	$\beta = 20^{\circ}$

Fig. 3 shows the differences in speed, drum motion (orbit), torque and mechanical power consumption regarding the static unbalanced load amounts in the empty drum.

From Fig. 3(a), it is seen that the mechanical frequency does not change even though the size of the unbalanced mass is different in the drum unless the location of the unbalanced load is not changed. Therefore, the peak-to-peak magnitude of the speed or torque signal within the periodic time is a key factor in determining the unbalanced mass. If the mechanical frequency or periodic time between the cycle on 100 rpm is changed depending on the unbalance size, then periodic time can be used for decision-making regarding the unbalanced load amount. The mechanical power can also be used if the periodic times among the unbalanced loads are not significant difference.

However, these approaches still have issues with detecting the unbalanced load size under the balanced load. For example, speed fluctuation or mechanical power fluctuation may not be identical even though the same unbalanced load is presented in the drum due to the effect of balanced loads. This means that in order to detect the unbalanced load size, the balanced load should be estimated beforehand to classify the amount of the unbalanced load.

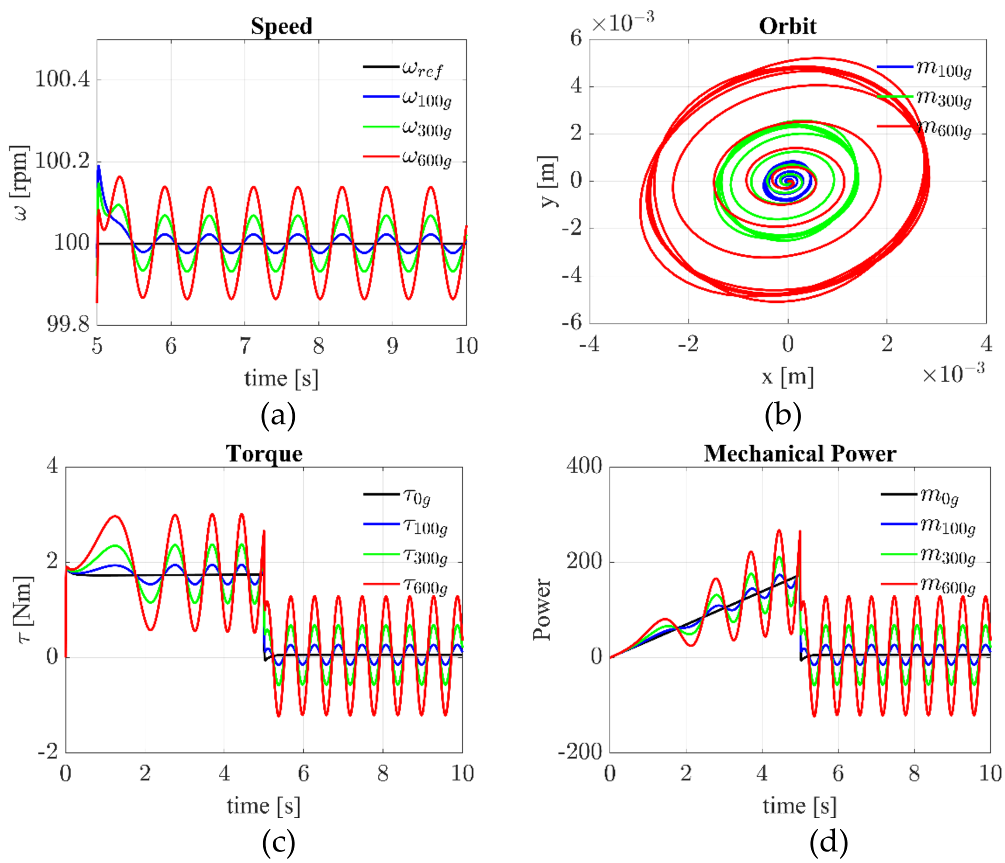

Figure 3 Washer Reponses with the Fixed Unbalanced Loads

Additionally, for the unbalanced load, we simulated the estimation process under two scenarios with the same balanced mass of $6\ kg$ and the fixed unbalance location of

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$\beta = 20^{\circ}$ from the drum position. Each simulation was performed with the same speed profile with every $1\ ms$.

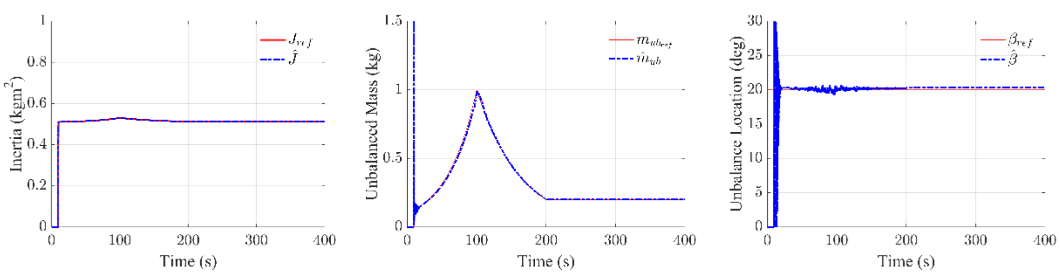
Case 1: Exponential Dynamic Unbalanced Load

For the first case, we mimic the change of the unbalanced load over time using the exponential function and apply the RLS algorithm to track the variation of the inertia and the unbalanced mass, while assuming the location of the unbalance is fixed.

Consider that dynamic unbalanced mass is formed after 10 seconds (t_1) before reaching the angular speed of 100 rpm and the laundry load is distributed and starts to stick to the drum wall with an exponentially increased unbalanced mass up to 1 kg. At the constant speed of 100 rpm, the water from the laundry is extracted so that the size of the unbalanced mass is decreasing exponentially up to 0.2 kg.

$$m_{ub} = \begin{cases} 0.1e^{0.023t} & for\ t \le t_1 \\ e^{-0.0161(t-t_1)} & for\ t_1 < t \le t_2 \\ 0.2 & for\ t_2 < t \end{cases} \tag{21}$$

Fig. 4 shows that the RLS is activated at 10 seconds and initial estimates have big errors due to the initial covariance matrix. However, Kalman gain matrix corrects the estimation error quickly so that the estimates are converged to the reference values.



(a) Inertia (b) unbalanced load (c) unbalance location

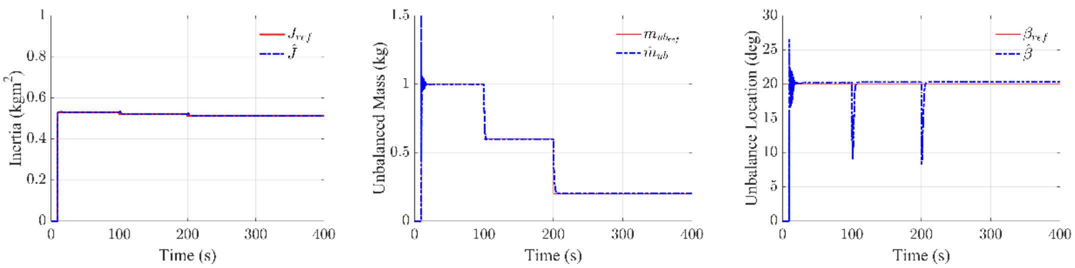
Figure 4 Estimation Results for Case 1

Case 2: Stair-Step Dynamic Unbalanced Load

Another perspective of the unbalanced variation during the drum rotation is that it changes in a steady state at the various speed plateau due to the different rate of the water extraction. In this reason, assume that the unbalanced load varies in a form of the stair-step starting from 1 kg to 0.2 kg, and become steady-state after 200 seconds.

$$m_{ub} = \begin{cases} 1 & for\ t \leq t_1 \\ 0.6 & for\ t_1 < t \leq t_2 \\ 0.2 & for\ t_2 < t \end{cases} \quad (22)$$

The results have similar trend of the ones in the exponential unbalance mass case.



(a) Inertia (b) unbalanced load (c) unbalance location

Figure 5 Estimation Results for Case 2

For both cases, the initial covariance matrix was chosen large enough ($P = 10^4 I$) with zero entries of the estimation vector at $k = 0$, and the forgetting factor was selected as $\lambda = 0.9995$ by the trial-and-error, which gives the best estimates for the inertia, unbalanced mass and its location. Table 2 shows the estimation accuracy evaluated by the root-mean-square-error (Δ = RMSE) after the RLS activation.

Table 2 Estimation Accuracy in RMSE

	Δ_{J_t}	$\Delta_{m_{ub}}$	Δ_{β}
Case 1	0.0012	0.1104	1.5739
Case 2	0.0018	0.1100	1.0496

4. Conclusion

In this study, to estimate the dynamic mechanical parameters for the horizontal washing machine in the simulation environment, we used the RLS approach with a forgetting factor. Balanced and unbalanced weights were considered in the mathematical model to better understand the dynamics of the washing machine. We chose the forgetting factor and initial covariance for the estimate procedure by taking into account the additional momentum caused by the varying imbalanced mass. Using the RLS, not only the variation of the unbalanced mass was estimated, but also the mechanical parameters such as the total inertia and the fixed location of the unbalanced mass were identified. From the study, the method has potential feasibility to identify the dynamic mechanical parameters simultaneously, while the conventional method requires separate algorithms to estimate the mechanical parameters.

For future work, we will implement the method in a motor control unit in practice.

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